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BPS Configuration of Supermembrane With Winding in M-direction

Kiyoshi EZAWA*[1], Yutaka MATSUO[2], Koichi MURAKAMI[3]

Yukawa Institute for Theoretical Physics
Kyoto University, Sakyo-ku, Kyoto 606-01, Japan

Abstract

We study de Wit-Hoppe-Nicolai supermembrane with emphasis on the winding in M-direction. We propose a SUSY algebra of the supermembrane in the Lorentz invariant form. We analyze the BPS conditions and argue that the area preserving diffeomorphism constraints associated with the harmonic vector fields play an essential role. We derive the first order partial differential equation that describes the BPS state with one quarter SUSY.

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*JSPS fellow
[1] e-mail address : ezawa@yukawa.kyoto-u.ac.jp
[2] e-mail address : yutaka@yukawa.kyoto-u.ac.jp
[3] e-mail address : murakami@yukawa.kyoto-u.ac.jp

1 Introduction

After the struggles to understand the still mysterious M theory, Matrix theory [1] emerged as the most successful candidate to describe the eleven dimensional theory. Although it has already passed many nontrivial tests, there remains nontrivial issues which needs careful examinations. One of such issues is the Lorentz invariance. Because of its very definition, Matrix theory needs the extra information to understand eleventh dimension (so called "M"-direction). Although there are some beautiful works [2] which suggest the symmetry by using 2+1 dimensional instanton calculus, it is still desirable to have a direct confirmation.

The situation is essentially different in its close cousin, de Wit-Hoppe-Nicolai (dWHN) supermembrane[3].[1] Although the difference between the two theories is simply in their gauge groups (SU(N) vs the area preserving diffeomorphism (APD)), we have an explicit definition of the Lorentz generators [5] and the Lorentz algebra itself was already checked explicitly [6][7].

In this letter, we examine the supermembrane in the toroidally compactified spacetime. In section two, we propose Lorentz invariant form of the SUSY algebra with the central charges associated with membranes. In section three, we derive the APD constraints associated with the harmonic vector fields which play a central role in the analysis of the BPS conditions. In sections four and five, we give equations that characterize BPS states with 1/2 and 1/4 SUSY. Examination of the latter gives a system of the first order differential equations which is analogous to the Bogomol'nyi bound of the super Yang-Mills theory. We show that a particular solution gives the BPS states of the type IIA superstring after the double dimensional reduction. Finally in section six we discuss how our results may be extended to the matrix formulation of M-theory.

2 Eleven Dimensional SUSY algebra of Supermembrane and BPS condition

Let us first examine the SUSY algebra of dWHN model. We use the same notations and definitions as in [5] in the following computation. In particular the expression of supercharges is given by:

$$\begin{aligned} Q^+ &= \frac{1}{\sqrt{P_0^+}} \int d^2\sigma \left(P^a \gamma_a + \frac{\sqrt{w}}{2} \{X^a, X^b\} \gamma_{ab} \right) \theta, \\ Q^- &= \sqrt{P_0^+} \int d^2\sigma \sqrt{w}\, \theta, \end{aligned} \tag{1}$$

where $\{A, B\} \equiv \frac{\epsilon^{rs}}{\sqrt{w}} \partial_r A \partial_s B$ $(r, s = 1, 2)$. Using the Dirac brackets:

$$\left(X^a(\sigma), P^b(\rho) \right)_{DB} = \delta^{ab} \delta^{(2)}(\sigma, \rho), \qquad \left(\theta_\alpha(\sigma), \theta_\beta(\rho) \right)_{DB} = -\frac{i}{\sqrt{w(\sigma)}} \delta_{\alpha\beta} \delta^{(2)}(\sigma, \rho),$$

[1] For a detailed information on the supermembrane theory, see [4] and references therein.

the SUSY algebra of dWHN model is computed as follows [3](see also [8]),

$$\begin{aligned}
i\left(Q_\alpha^-, Q_\beta^-\right)_{DB} &= \delta_{\alpha\beta} P_0^+, \\
i\left(Q_\alpha^-, Q_\beta^+\right)_{DB} &= P_0^a (\gamma_a)_{\alpha\beta} + \frac{1}{2} z^{ab} (\gamma_{ab})_{\alpha\beta}, \\
i\left(Q_\alpha^+, Q_\beta^+\right)_{DB} &= 2\delta_{\alpha\beta} H + 2 z^a (\gamma_a)_{\alpha\beta} + \frac{2}{4!} z^{abcd} (\gamma_{abcd})_{\alpha\beta}. \qquad (2)
\end{aligned}$$

The brane charges which appear in the right hand side of these equations are defined by

$$\begin{aligned}
z^{ab} &= -\int d^2\sigma \sqrt{w} \left\{X^a, X^b\right\}, \qquad (3) \\
z^a &= \frac{1}{P_0^+} \int d^2\sigma \left(\left\{X^a, X^b\right\} P_b - \frac{i}{2} \sqrt{w} \left\{X^a, \theta^\alpha\right\} \theta^\alpha \right) \\
&\quad - \frac{3i}{16 P_0^+} \int d^2\sigma \sqrt{w} \left\{X^c, \theta\gamma^{ac}\theta\right\}, \qquad (4) \\
z^{abcd} &= -\frac{12}{P_0^+} \int d^2\sigma \sqrt{w} \left\{X^{[a}, X^b\right\} \left\{X^c, X^{d]}\right\} - \frac{i}{4P_0^+} \int d^2\sigma \sqrt{w} \left\{X^{[a}, \theta\gamma^{bcd]}\theta\right\}. \qquad (5)
\end{aligned}$$

The second term in (4) and the second term in (5) should vanish as we already discussed in our previous paper [7] (appendix F) to make the supercharge well-defined. The first term in (5) vanishes for the membrane configuration. (5) should be regarded as the longitudinal 5 brane charge but it becomes absent in the supermembrane. Finally, the first term in (4) can be rewritten as

$$\int d^2\sigma \sqrt{w} \left\{X^-, X^a\right\}. \qquad (6)$$

It makes the SUSY algebra (2) Lorentz invariant[2].

In [8], the BPS conditions of the SUSY algebra was discussed in the Matrix theory. It is our purpose here to reexamine the analysis for the manifestly Lorentz invariant form (2). We write the SUSY algebra in the matrix form,

$$\begin{aligned}
\begin{pmatrix} i\left(Q^-, Q^-\right)_{DB} & i\left(Q^-, Q^+\right)_{DB} \\ i\left(Q^+, Q^-\right)_{DB} & i\left(Q^+, Q^+\right)_{DB} \end{pmatrix} &= \begin{pmatrix} P_0^+ \cdot I_{16} & \mathbf{P} + \mathbf{z}_2 \\ \mathbf{P} - \mathbf{z}_2 & 2H \cdot I_{16} + 2\mathbf{z}_1 \end{pmatrix} \\
= \begin{pmatrix} P_0^+ \cdot I_{16} & 0 \\ \mathbf{P} - \mathbf{z}_2 & I_{16} \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{P_0^+} I_{16} & 0 \\ 0 & \frac{1}{P_0^+} m \end{pmatrix} &\cdot \begin{pmatrix} P_0^+ \cdot I_{16} & \mathbf{P} + \mathbf{z}_2 \\ 0 & I_{16} \end{pmatrix}. \qquad (7)
\end{aligned}$$

Here our notation is $\mathbf{P} = P_0^a \gamma_a$, $\mathbf{z}_1 = z^a \gamma_a$, $\mathbf{z}_2 = \frac{1}{2} z^{ab} \gamma_{ab}$. The real symmetric matrix m is defined as,

$$\begin{aligned}
m &= 2P_0^+ (H \cdot I_{16} + \mathbf{z}_1) - (\mathbf{P} - \mathbf{z}_2)(\mathbf{P} + \mathbf{z}_2) \\
&= (2P_0^+ \cdot H - P_0^a P_0^a - \frac{1}{2} z^{ab} z^{ab}) I_{16} + 2(P_0^+ z^a - P_0^c z^{ca}) \gamma_a \\
&\quad + \frac{1}{4} z^{ab} z^{cd} \gamma^{abcd}. \qquad (8)
\end{aligned}$$

[2] We understand the SUSY algebra in this form was also derived by de Wit et. al [9]. We thank B. de Wit to send us the preliminary version of their paper. We have to admit that some part of this paper have overlaps with theirs although it was studied independently.

From (7) we find that m is positive semi-definite when the theory is quantized.

At this point, it is easy to observe that the BPS condition of 1/2 SUSY is simply $m = 0$ and that of 1/4 SUSY is that m has rank 8. We will analyze these conditions in detail in sections 4 and 5.

3 Constraint from APD

Associated with the gauge symmetry in the 0+1 dimensional Yang-Mills system, the Gauss law constraint of the dWHN model is given by

$$\begin{aligned} \varphi(\sigma) &= -\left\{\frac{P^a}{\sqrt{w}}, X^a\right\} - \frac{i}{2}\{\theta,\theta\} \approx 0 \qquad (9)\\ \varphi^{(\lambda)} &= \int d^2\sigma \epsilon^{rs}\phi_r^{(\lambda)}\left(P_0^+\partial_s X^- + \frac{P^a}{\sqrt{w}}\partial_s X^a + \frac{i}{2}\theta\partial_s\theta\right) \approx 0. \qquad (10)\end{aligned}$$

The first constraint comes from the area preserving diffeomorphism (APD) in the bulk. The second ones are associated with the harmonic one form $\phi_r^{(\lambda)}$ where $\lambda = 1, \cdots, 2g$ (g is the genus of the surface). These two conditions ensure the integrability of the definition of X^-,

$$\partial_r X^-(\sigma) = -\frac{1}{P_0^+}\left(\frac{P^a}{\sqrt{w}}\partial_r X^a + \frac{i}{2}\theta\partial_r\theta\right). \qquad (11)$$

When the target space has a toroidal topology,

$$X^a \sim X^a + 2\pi R^a, \quad X^- \sim X^- + 2\pi R, \qquad (12)$$

and the membrane has certain winding number, the embedding coordinates and their momenta can be expanded in terms of the eigenfunction of the Laplacian as follows,[3]

$$\begin{aligned} \partial_r X^a(\sigma) &= 2\pi R^a \phi_r^{(\lambda)} n^{(\lambda)a} + \sum_A X_A^a \partial_r Y^A(\sigma),\\ \partial_r X^-(\sigma) &= 2\pi R\phi_r^{(\lambda)} n^{(\lambda)} + \sum_A X_A^- \partial_r Y^A(\sigma),\\ P^a(\sigma) &= P^+(\sigma)\frac{\partial}{\partial t}X^a(\sigma) = \sqrt{w}\left(\frac{m^a}{R^a} + \sum_A P_A^a Y^A(\sigma)\right),\\ P^+(\sigma) &= \sqrt{w}P_0^+ = \sqrt{w}\frac{m}{R},\\ \theta^\alpha(\sigma) &= \theta_0^\alpha + \sum_A \theta_A^\alpha Y^A(\sigma). \qquad (13)\end{aligned}$$

[3] We normalize the harmonic one forms $\phi_r^{(\lambda)}$ as

$$\oint_{C^{\lambda'}} d\sigma^r \phi_r^{(\lambda)} = \delta^{\lambda\lambda'},$$

where C^λ ($\lambda = 1, 2, \ldots, 2g$) comprize a basis of the first homology class.

Here Y_A is the eigenfunction of the Laplacian with non-zero eigenvalue, $\Delta Y_A = -\omega_A Y_A$, $\omega_A > 0$. $n^{(\lambda)a}$, $n^{(\lambda)}$, m^a and m are integer-valued. We plug the expansion into (10) to get

$$\begin{aligned}\varphi^{(\lambda)} &= 2\pi f_{\lambda\lambda' 0}(mn^{(\lambda')} + m^a n^{(\lambda')a}) + 2\pi \sum_{\lambda', B} f_{\lambda\lambda'}{}^{B} R^a n^{(\lambda')a} P^a_B \\ &\quad + \sum_{AB} f_\lambda{}^{AB}(X^a_A P^a_B - \frac{i}{2}\theta_A\theta_B).\end{aligned} \tag{14}$$

The structure constants are defined as

$$f_{\lambda AB} = \int d^2\sigma \epsilon^{rs}\phi^{(\lambda)}_r \partial_s Y_A Y_B, \quad f_{\lambda\lambda' B} = \int d^2\sigma \epsilon^{rs}\phi^{(\lambda)}_r \phi^{(\lambda')}_s Y_B.$$

In our analysis in the following sections, we mainly take the topology of the membrane as two torus. If we pick the coordinate σ^r to satisfy $\sigma^r \sim \sigma^r + 1$ $(r = 1, 2)$, the eigenfunction becomes $Y^A = e^{2\pi i(A_1\sigma^1 + A_2\sigma^2)}$ with $A = (A_1, A_2) \neq (0, 0)$, $A_i \in \mathbf{Z}$. We write the expansion of the embedding function as,

$$\begin{aligned}X^- &= -\frac{R}{m}Ht + 2\pi R n_r \sigma^r + \hat{X}^-(\sigma) \\ X^a &= \frac{R}{m}\frac{m^a}{R^a}t + 2\pi R^a n^a_r \sigma^r + \hat{X}^a(\sigma),\end{aligned} \tag{15}$$

where the periodic parts are given by $\hat{X}^\mu(\sigma) = \sum_A X^\mu_A Y^A$, and so on.

The central charges of the toroidal membrane are given as

$$\begin{aligned}z^{ab} &= -(2\pi)^2 R^a R^b (n^a_1 n^b_2 - n^a_2 n^b_1) \\ z^a &= (2\pi)^2 R R^a (n_1 n^a_2 - n_2 n^a_1).\end{aligned} \tag{16}$$

The constraint (10) is simplified as,

$$2\pi\varphi_r \equiv -\epsilon_{rs}\varphi^{(s)} = 2\pi\left(mn_r + m^a n^a_r + i\sum_{A\neq\vec{0}} A_r(P^a_{-A}X^a_A + \frac{i}{2}\theta^\alpha_{-A}\theta^\alpha_A)\right) \approx 0. \tag{17}$$

As we will see, this condition may be regarded as an analogue of the level matching condition in string theory.

4 BPS configuration with 1/2 SUSY

In the following we will mainly consider the case when the topology of the membrane is two torus. In such situation, the last term in (8) vanishes which facilitates the analysis of the BPS condition.

By using the definition of the invariant supermembrane mass $\mathcal{M}$:

$$\mathcal{M}^2 = 2P^+_0 \cdot H - P^a_0 P^a_0, \tag{18}$$

the BPS condition $m = 0$ becomes[4],

$$\begin{aligned} \mathcal{M}^2 &= \frac{1}{2} z^{ab} z^{ab} \\ P_0^+ z^a - P_0^c z^{ca} &= 0. \end{aligned} \tag{19}$$

The second condition relates the winding in the longitudinal direction to those in the transverse dimensions.

Now we can discuss the relationship between (17) and the BPS condition (19). The first equation in (19) tells us that there is no nonzero mode contribution. This implies

$$\begin{aligned} X^a &= \frac{R}{R^a}\frac{m^a}{m} t + 2\pi R^a n_r^a \sigma^r, \\ X^- &= -\frac{R}{m} H t + 2\pi R n_r \sigma_r, \\ \theta^\alpha &= \theta_0^\alpha. \end{aligned} \tag{20}$$

The constraint (17) is reduced to the following simple relation

$$\varphi_r^{(0)} \equiv m n_r + m^a n_r^a = 0. \tag{21}$$

Using this, the second equation of (19) is rewritten as

$$\varphi_1^{(0)} n_2^a - \varphi_2^{(0)} n_1^a = 0. \tag{22}$$

We therefore conclude that, by virtue of the constraint (17), the 1/2 BPS condition (19) is automatically satisfied even for the membrane wrapping in the M-direction (with no nonzero modes). This suggests that (17) plays an important role in showing the Lorentz invariance of the supermembrane theory.

5 BPS configurations with 1/4 SUSY

Let us proceed to explore the equation of supermembrane with one quarter SUSY. As before we assume the toroidal topology of the supermembrane. The BPS condition is that the matrix m in (8) has rank 8. Since $P_0^+ z^a - P_0^c z^{ca}$ is a constant vector, we can always choose a nine-dimensional orthonormal basis $(e_a^{(9)}, e_a^{(i)})$ $(i = 1, 2, \ldots, 8)$ with the property

$$P_0^+ z^a - P_0^c z^{ca} \propto e^{(9)a}. \tag{23}$$

We will henceforth denote the components of a vector V^a as

$$V^9 = e_a^{(9)} V^a, \quad V^i = e_a^{(i)} V^a. \tag{24}$$

In this frame, the BPS condition is equivalent to

$$\mathcal{M}^2 - \frac{1}{2} z^{ab} z^{ab} \mp 2(P_0^+ z^9 - P_0^c z^{c9}) = 0. \tag{25}$$

[4]We point out that $\mathcal{M}^2$ is Lorentz invariant by virtue of $z^{a+} = 0$.

We introduce the notation, $\nabla^a \equiv 2\pi R^a (n_1^a \partial_2 - n_2^a \partial_1)$. Various parts in the Hamiltonian are written as,

$$
\begin{aligned}
\left\{X^a, X^b\right\} &= -z^{ab} + \nabla^a \hat{X}^b - \nabla^b \hat{X}^a + \left\{\hat{X}^a, \hat{X}^b\right\}, \\
\left\{X^a, P^b\right\} &= \nabla^a \hat{P}^b + \left\{\hat{X}^a, \hat{P}^b\right\}.
\end{aligned} \tag{26}
$$

In the following analysis, we assume the fermionic background to vanish for simplicity. The left hand side of (25) becomes,

$$
\begin{aligned}
& \int d^2\sigma \left(\hat{P}^a \hat{P}^a + \frac{1}{2} (\nabla^a \hat{X}^b - \nabla^b \hat{X}^a + \left\{\hat{X}^a, \hat{X}^b\right\})^2 \right) \mp 2 \int d^2\sigma \hat{P}^c \nabla^9 \hat{X}^c \\
= & \int d^2\sigma \left[\left(\hat{P}^c \mp (\nabla^9 \hat{X}^c - \nabla^c \hat{X}^9 + \left\{\hat{X}^9, \hat{X}^c\right\}) \right)^2 \right. \\
& \left. + \frac{1}{2} \left(\nabla^i \hat{X}^j - \nabla^j \hat{X}^i + \left\{\hat{X}^i, \hat{X}^j\right\} \right)^2 \right] \geq 0.
\end{aligned} \tag{27}
$$

In deriving this equation, we used the APD constraints (9) (10). The final expression becomes a sum of the squares as expected. The BPS condition for 1/4 SUSY becomes,

$$
\begin{aligned}
\hat{P}^9 &= 0, \\
\hat{P}^i &= \pm \left(\left\{X^9, X^i\right\} + z^{9i} \right), \\
0 &= \left\{X^i, X^j\right\} + z^{ij}.
\end{aligned} \tag{28}
$$

This equation[5] is an analogue of the Bogomol'nyi bound of the super Yang-Mills theory (see for example [11]). We note that, in the situation considered here, the following equation holds

$$
P_i^+ z^i - P_0^c z^{ci} = 0. \tag{29}
$$

The SUSY generators which are not broken under such a configuration are,

$$
\begin{aligned}
Q^{(\mp)} &\equiv \Pi^\mp \left\{ \sqrt{P_0^+} Q^+ - \frac{(\mathbf{P} - \mathbf{z}_2)}{\sqrt{P_0^+}} Q^- \right\} \\
&= \Pi^\mp \int d^2\sigma \left(\hat{P}^a \gamma^a + \frac{1}{2} \left(\left\{X^a, X^b\right\} + z^{ab} \right) \gamma_{ab} \right) \hat{\theta},
\end{aligned} \tag{30}
$$

where $\Pi^\mp = (1 \mp \gamma^9)/2$ are projection operators. In fact these generators have vanishing Dirac brackets with canonical variables, e.g.,

$$
\begin{aligned}
\left(Q^{(\mp)}, \theta \right)_{DB} &= -i \Pi^\mp \left[\mp \hat{P}^9 + \left(\hat{P}^i \mp \left(\left\{X^9, X^i\right\} + z^{9i} \right) \right) \gamma_j + \frac{1}{2} \left(\left\{X^i, X^j\right\} + z^{ij} \right) \gamma_{ij} \right] \\
&= 0.
\end{aligned} \tag{31}
$$

We remark that, in general, the right hand side of (31) need not be strictly zero. It is sufficient to set it zero modulo APD gauge transformations. This enables us to analyze the case of non-vanishing θ.

[5]Similar problem was approached by Becker, Becker and Strominger [10] in a slightly different context.

As an illustration let us consider the following configuration:

$$
\begin{aligned}
X^9 &= \frac{R}{R^9}\frac{m^9}{m}t + 2\pi R^9 n^9 \sigma^2, \\
X^i &= \frac{R}{R^i}\frac{m^i}{m}t + 2\pi R^i n^i \sigma^1 + \hat{X}^i(t,\sigma^1), \\
\theta^\alpha &= \theta_0^\alpha + \hat{\theta}^\alpha(t,\sigma^1), \\
X^- &= -\frac{R}{m}Ht + 2\pi R(n\sigma^1 + n'\sigma^2) + \hat{X}^-(t,\sigma^1), \\
P^+ &= \frac{m}{R}.
\end{aligned} \tag{32}
$$

This configuration has the central charges

$$
z^9 = 4\pi^2 RR^9 nn^9, \quad z^i = -4\pi^2 RR^i n' n^i, \quad z^{9i} = 4\pi^2 R^9 R^i n^9 n^i, \quad z^{ij} = 0. \tag{33}
$$

The Gauss law constraint for this configuration reduces to[6]

$$
\begin{aligned}
\varphi_1 &= mn + m^i n^i + \frac{1}{2\pi}\int d\sigma^1 (\hat{P}^i \partial_1 \hat{X}^i + \frac{i}{2}\hat{\theta}\partial_1\hat{\theta}) \approx 0, \\
\varphi_2 &= mn' + m^9 n^9 \approx 0.
\end{aligned} \tag{34}
$$

The first equation is of the same form as the level-matching condition of the closed superstring. This is consistent with the fact that, after the double dimensional reduction, 11D supermembrane reduces to 10D type IIA superstring [12]. The BPS condition is rewritten as

$$
\begin{aligned}
\partial_t \hat{X}^i &= \mp 2\pi RR^9 \frac{n^9}{m}\partial_1 \hat{X}^i, \\
\Pi^\pm \hat{\theta} &= 0.
\end{aligned} \tag{35}
$$

The second equation comes from the condition: $(Q^{(\mp)}, X^a)_{DB} = 0$ *mod APD*. It leads us to see that the fermion modes with plus (minus) chirality are projected out. Combined with equations of motion, the condition (35) picks up only the left(right)-handed modes in the σ^1-direction. These configurations are therefore understood as an extension of the BPS configurations in the type IIA superstring to 11D supermembrane.

6 Discussion

In this paper we investigated winding modes of the supermembrane in the light cone gauge. We have obtained the following results: (i) 1/2 SUSY is achieved even if the membrane wraps around the longitudinal direction; (ii) success in constructing such configurations is attributed to the Gauss law constraint (10) associated with the harmonic vector fields; (iii) we derive the first order differential equations to characterize 1/4 SUSY; (iv) we explicitly constructed string BPS states from those of the membrane.

[6] The constraint in the bulk, $\varphi(\sigma) \approx 0$, is automatically satisfied in this case.

While the constraint (10) has been overlooked in the previous analysis of M(atrix) theory, it may play an essential role if our result is taken seriously. Thus it may be useful to consider an extension of (10) to M(atrix) theory. In the case of a toroidal supermembrane, we can construct an obvious candidate:

$$\varphi_M^{(1)} = 2\pi m n^2 + \mathrm{Tr}\left([q, X^a]P^a - \frac{i}{2}[q, \theta^\alpha]\theta^\alpha\right),$$
$$\varphi_M^{(2)} = -2\pi m n^1 + \mathrm{Tr}\left([p, X^a]P^a - \frac{i}{2}[p, \theta^\alpha]\theta^\alpha\right). \tag{36}$$

X^a, P^a and θ^α are now regarded as $m \times m$ matrix-valued and (q, p) are the matrices with the commutation relation $[q, p] = I$. A candidate for the longitudinal membrane in M(atrix) theory is also obtained if we replace (σ^1, σ^2) in (20) by (q, p).

One important point is that the generalization of the 1/4 condition to the M(atrix) theory is straightforward. All we have to do is to replace the APD bracket with the commutator.

We hope that our approach gives a new viewpoint to this famous problem and the relation with the BPS membrane state in the supergravity theory [13] will be very interesting.

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